(1) Use the Euclidean algorithm to find gcd(451, 369). (10 points)

(2) Let \( a, b, c \in \mathbb{Z} \). Show that
(a) \( \gcd(ab, c) \mid \gcd(a, c) \gcd(b, c) \);
(b) \( \gcd(ab, c) = \gcd(a, c) \gcd(b, c) \) if \( a \perp b \). (10+10 points)

(3) Let \( m, n \geq 0 \). Show that \( \gcd(2^m - 1, 2^n - 1) = 2^{\gcd(m,n)} - 1 \).
Hint. Assuming \( m \geq n \), show that \( \gcd(2^m - 1, 2^n - 1) = \gcd(2^{m-n} - 1, 2^n - 1) \). (20 points)

(4) Let \( F_n = 2^{2^n} + 1, n \geq 0 \), be the \( n \)th Fermat number. Show that the Fermat numbers are coprime in pairs, and deduce that there are infinitely many primes.
Hint. If \( m < n \), then \( F_m \mid F_n - 2 \). (20 points)

(5) (a) Assume that \( a > b > 0 \). Prove that the Euclidean Algorithm finds \( \gcd(a, b) \) after at most \( 1 + C \log b \) iterations of the loop, where \( C \) is some real number.
Hint. Consider two iterations at a time.
(b) Bonus Problem
Find the best possible constant \( C \) when \( b \to \infty \): determine
\[
\limsup_{b \to \infty} \frac{C(b)}{\log b}
\]
where \( C(b) \) is the maximal number of iterations required to compute \( \gcd(a, b) \)
for some \( a \). (20+15 points)

(6) Let \( a, b \) be positive coprime integers.
(a) Show that any sufficiently large integer \( n \) can be written as \( n = ax + by \)
with nonnegative integers \( x \) and \( y \).
(b) Bonus Problem
What is the largest \( n \) that \( cannot \) be represented in this way? (10+15 points)